Nonlocal charge transport mediated by spin diffusion in the spin Hall effect regime

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A nonlocal electric response in the spin Hall regime, resulting from spin diffusion mediating charge conduction, is predicted. The spin-mediated transport stands out due to its long-range character, and can give dominant contribution to nonlocal resistance. The characteristic range of nonlocality, set by the spin diffusion length, can be large enough to allow detection of this effect in materials such as GaAs despite its small magnitude. The detection is facilitated by a characteristic nonmonotonic dependence of transresistance on the external magnetic field, exhibiting sign changes and decay.

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I. INTRODUCTION

The spin Hall effect (SHE) is a phenomenon arising due to spin-orbit coupling in which charge current passing through a sample leads to spin transport in the transverse direction.[1] This phenomenon has been attracting continuous interest, partially because of the rich physics and diversity of SHE mechanisms[2] and partially because SHE enables generating spin polarization on a micron scale and electrical detection of spin-polarized currents, which are the key ingredients of spintronics.[3] Theoretically, two main types of SHE have been studied: (i) extrinsic, being due to spin-dependent scattering on impurities[1] and (ii) intrinsic, arising from the spin-orbit terms in the band Hamiltonian.[4] Both extrinsic and intrinsic SHE have been detected experimentally[5–8] using optical techniques. Reciprocal SHE (that is, transverse voltage induced by a spin-polarized current) was observed in Al nanowire,[9] where ferromagnetic contacts were used to inject spin-polarized current into the sample, and in Pt film.[10]

Here we show that the SHE relation between charge current and spin current leads to an interesting spin-mediated nonlocal charge transport, in which spins generated by SHE diffuse through the sample and, by reverse SHE, induce electric current elsewhere. The range of nonlocality of this charge transport mechanism is on the order of the spin diffusion length $\ell_s = \sqrt{D_s \tau_s}$, where D_s is the spin diffusion constant, and τ_s is the spin relaxation time. The observation of such nonlocal charge transport due to SHE can be made fully electrically, which represents a distinct advantage compared to the methods relying on the sources of spin-polarized current.[9,10] Although the nonlocal electrical signal, estimated below, is small, by optimizing the multiterminal geometry one can enhance the nonlocal character of the effect and easily distinguish it from the Ohmic transport.

The main distinction of the spin-mediated electrical effect considered in the present work from related ideas discussed earlier[11–14] is that here we identify a situation in which the spin-mediated charge transport, due to its nonlocality, *dominates* over the Ohmic contribution. In particular, Refs. 11 and 14 considered a correction to the bulk conductivity resulting from spin diffusion and SHE, detectable by its characteristic contribution to magnetoresistance. In Ref. 13 a spin-assisted electrical response was predicted in a multiterminal mesoscopic system, studied numerically in a weak disorder regime with the mean free path comparable to the system size.

In this paper we shall analyze systems with short mean free path relevant for current experimental investigations,[5,7–10] focusing mostly on the extrinsic SHE regime. The geometry that will be of interest to us is a strip of width w narrow compared to the spin diffusion length ℓ_s, with current and voltage probes attached as illustrated in Fig. 1. The nonlocal charge transport gives rise to transresistance: a voltage V_{12} across the strip measured at distances $x \sim \ell_s$ from the current source (S) and drain (D). exponentially on the length scale w/π away from the source, the spin-dependent voltage can easily exceed the spin-independent contribution. This is in contrast to Refs. 11, 13, and 14 where the predicted spin-dependent fraction of conductance is small.

In addition, the proposed setup allows one to investigate interesting spin transport phenomena predicted in the intrinsic SHE regime: the drift-induced spin precession[15] and quenching of spin relaxation in quasi-one-dimensional (quasi-1D) systems.[16] As discussed below, these phenomena are directly manifest in the transresistance.

Below we demonstrate that the spin-mediated nonlocal effect exhibits a characteristic oscillatory dependence on the in-plane magnetic field resulting from spin precession. At a distance $|x| \sim \ell_s$ from the source it will oscillate and decay on

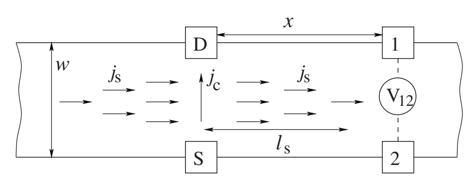

FIG. 1. Nonlocal spin-mediated charge transport schematic. Charge current j_c applied across a narrow strip generates, via SHE, a longitudinal spin current j_s. After diffusing over a distance $x \sim \ell_s \gg w$, the spins induce a transverse voltage V_{12} on probes 1 and 2 via the reciprocal SHE. For a narrow strip, $w \ll \ell_s$, the spin-mediated nonlocal contribution exceeds the Ohmic contribution that decays as $e^{-\pi|x|/w}$.

the magnetic field scale $\omega_B \sim 1/\tau_s$, where ω_B is the electron spin Larmor frequency. This oscillatory field dependence can be used in experiment to extract parameters such as the spin diffusion length and spin Hall coefficient.

The rest of the paper is organized as follows. In Sec. II we introduce our transport model, find the spin and charge current densities, and obtain an analytic expression for the nonlocal resistance. In Sec. III we use this analytic result to estimate the magnitude of the nonlocal resistance for several materials that exhibit extrinsic SHE. In Sec. IV we study the dependence of the nonlocal resistance on the in-plane magnetic field. The nonlocal transport for materials with intrinsic SHE is briefly discussed in Sec. V. Finally, Sec. VI summarizes our results.

II. MODEL

We consider, as a simplest case, an infinite narrow strip,

$$-\infty < x < +\infty, \quad -w/2 < y < w/2 \quad (w \ll \ell_s),$$

as illustrated in Fig. 1. We assume, without loss of generality, that current source and drain leads are narrow, connected to the sample at the points $(0, \pm\frac{1}{2}w)$.

The electric potential in such a sample, described by Ohmic conductivity σ, can be found as a solution of the two-dimensional (2D) Laplace equation with the boundary condition $j_y(x, y=\pm\frac{1}{2}w)=I\delta(x)$, where I is the external current. Solving it by the Fourier method, we find

$$\varphi(x,y) = -\int dk \frac{Ie^{ikx}\sinh(ky)}{2\pi\sigma k\cosh\left(\frac{1}{2}kw\right)}. \quad (1)$$

In what follows we will need the electric field tangential component at the boundaries of the strip, $E_{x,\pm}(x) = -\partial_x\varphi_\pm(x)$, where + and − signs correspond to the strip upper edge $(y=+w/2)$ and lower edge $(y=-w/2)$. This component of the electric field is found from potential (1) at the boundaries of the sample as

$$E_{x,\pm}(x) = \mp\frac{2I\,\mathrm{sgn}\,x}{w\sigma}\sum_{\mathrm{odd}\ n>0} e^{-n|\tilde{x}|} = \mp\frac{I}{w\sigma\sinh\tilde{x}}, \quad (2)$$

where $\tilde{x}=\pi x/w$. Potential difference between the strip edges, $\Delta\varphi=\varphi_+(x)-\varphi_-(x)$, evaluated in a similar way, decreases as $e^{-\pi|x|/w}$ at $|x|\gtrsim w$.

Below we focus on the case of extrinsic SHE, when the k-linear Dresselhaus and Rashba terms in electron spectrum are negligible. Then the spin fluctuation generated by SHE evolves according to the diffusion equation,

$$D_s\partial^2 s(x,y) - \Xi(x,y) - \frac{1}{\tau_s}s(x,y) = 0, \quad (3)$$

where s is the z component of the spin density, D_s is the spin diffusion coefficient, and τ_s is the spin relaxation time. The source term Ξ in Eq. (3) describes spin current arising due to the spin Hall effect,

$$\Xi_\gamma(x,y) = \nabla\cdot j_s = \partial_\alpha[\beta_s\varepsilon_{\alpha\beta\gamma}E_\beta(x,y)], \quad (4)$$

where β_s is the spin Hall conductivity. In the presence of the k-linear Dresselhaus and/or Rashba interaction, spin transport is more complicated due to spin-orbit (SO) induced precession and dephasing.[17,18] The modification of the nonlocal electric effect in this case will be briefly discussed at the end of the paper.

Since $\nabla\times\mathbf{E}=0$, the spin current source [Eq. (4)] vanishes in the bulk and is only nonzero at the strip boundaries,

$$\Xi(x,y) = \beta_s\delta\left(y-\frac{1}{2}w\right)E_{x,+}(x) - \beta_s\delta\left(y+\frac{1}{2}w\right)E_{x,-}(x). \quad (5)$$

The distinction from the Ohmic contribution becomes most clear when our strip is relatively narrow, $w\ll\ell_s$. In this case the spin current and spin density induced by SHE are approximately constant across the strip. Thus we can integrate over y and solve a one-dimensional spin diffusion problem. Suppressing the y dependence in Eq. (3), we take $\Xi(x) = \beta_s E_{x,+}(x) - \beta_s E_{x,-}(x)$.

Solution of Eq. (3) in the Fourier representation reads,

$$s_k = -\frac{p_k}{D_s k^2 + 1/\tau_s}, \quad p_k = \frac{1}{2\pi}\int_{-\infty}^{+\infty} dx\,\Xi(x)e^{-ikx}. \quad (6)$$

This expression can be simplified by noting that $\Xi(x)$ is an odd function of x and that the integral over x converges at $|x|\lesssim w$, while we are interested in the harmonics with much lower $k\sim 1/\ell_s \ll 1/w$. Sending k to zero in the integral, we obtain

$$s_k = \frac{1}{\pi}\frac{ikG}{D_s k^2 + 1/\tau_s}, \quad (7)$$

where the spin dipole G is given by

$$G = \int_0^\infty \Xi(x)x\,dx = -\frac{I\beta_s w}{2\sigma} \quad (8)$$

[we used Eqs. (2) and (5) to evaluate this expression].

Now we can find the spin current

$$J_s(x) = -D_s\partial_x s(x), \quad (9)$$

where the spin density $s(x)$ is obtained by the inverse Fourier transform of Eq. (7). Using Eq. (8), we find

$$J_s(x) = \frac{I\beta_s w}{2\sigma\ell_s}e^{-|x|/\ell_s}. \quad (10)$$

(this expression is valid for x not too close to the source, $|x|\gtrsim w$). Expression (10) gives the net spin current rather than the spin current density, as we have been solving a 1D diffusion problem. This spin current generates, by reverse SHE, a voltage across the sample,

$$\delta V(x) = \frac{\beta_c J_s(x)}{\sigma} = \frac{I\beta_c\beta_s w}{2\ell_s\sigma^2}e^{-|x|/\ell_s}, \quad (11)$$

where β_c describes charge current arising in response to spin current, $j_c^\alpha = \beta_c\varepsilon_{\alpha\beta}j_s^\beta$. Relating β_c to the spin Hall conductivity as $\beta_c=\beta_s/\sigma$, we write the nonlocal response [Eq. (11)] as a transresistance,

$$R_{\mathrm{nl}}(x)=\frac{\delta V(x)}{I}=\frac{1}{2}\left(\frac{\beta_s}{\sigma}\right)^2\frac{w}{\sigma l_s}e^{-|x|/\ell_s}. \qquad (12)$$

We emphasize that for the extrinsic SHE,[1] the spin current is established on the length scale of the order of the electron mean free path ℓ, here taken to be much smaller than the strip width w. Thus the inhomogeneity of the charge current j_c (Fig. 1) on the length scale set by w does not affect our analysis. The estimate [Eq. (12)] for the nonlocal voltage is therefore accurate as long as $w \gg \ell$.

III. ESTIMATES OF THE NONLOCAL EFFECT FOR VARIOUS MATERIALS

We now compare the magnitude of the nonlocal contribution [Eq. (12)] for several materials where extrinsic SHE has been observed. For the transresistance [Eq. (12)] to be large, one would like to have a material with a large ratio β_s/σ, and a large spin diffusion length ℓ_s. For Si-doped GaAs with electron density $n=3\times10^{16}$ cm^{-3}, the three-dimensional (3D) charge conductivity, spin Hall conductivity, and spin diffusion length, reported in Refs. 5 and 7, are given by $\sigma_{3D}\approx 2.5\times10^{-3}$ Ω^{-1} μm^{-1}, $\beta_{s3D}\approx 5\times10^{-7}$ Ω^{-1} μm^{-1}, and $\ell_s\approx 9$ μm. Our two-dimensional quantities σ and β_s are related to the 3D quantities as $\sigma=\sigma_{3D}w_z$ and $\beta_s=\beta_{s3D}w_z$, where w_z is the sample thickness. Taking $w_z=2$ μm,[5,7] and choosing the sample width to be $w=0.5$ μm, we estimate the transresistance [Eq. (12)] as

$$R_{\mathrm{nl}}(x)\approx 2\times10^{-7}\times e^{-|x|/\ell_s}\ (\Omega). \qquad (13)$$

Although small, it by far exceeds the Ohmic conduction contribution which at a distance x is proportional to $\sigma^{-1}e^{-\pi|x|/w}$. Indeed, for a typical $x\approx\ell_s$ the Ohmic contribution is negligibly small: $e^{-\pi\ell_s/w}\approx e^{-57}\approx 10^{-24.8}$.

In the case of InGaAs, the 3D charge conductivity and 3D spin Hall conductivity have values similar to those quoted above for GaAs (see Ref. 5), $\sigma_{3D}\approx 2.5\times10^{-3}$ Ω^{-1} μm^{-1} and $\beta_{s3D}\approx 5\times10^{-7}$ Ω^{-1} μm^{-1}, while spin diffusion length is considerably shorter, $\ell_s\approx 2$ μm. Therefore, in order for the nonlocal voltage [Eq. (12)] at $|x|\sim\ell_s$ to exceed the Ohmic contribution, proportional to $\sigma^{-1}e^{-\pi|x|/w}$, the sample width w must satisfy $w\ll\pi\ell_s/[2\ln(\sigma_{3D}/\beta_{3D})]\approx 360$ nm.

Another material exhibiting extrinsic SHE is ZnSe.[8] For carrier concentration $n=9\times10^{18}$ cm^{-3} the 3D charge and spin Hall conductivities are given by $\sigma_{3D}\approx 2\times10^{-1}$ Ω^{-1} μm^{-1} and $\beta_{s3D}\approx 3\times10^{-6}$ Ω^{-1} μm^{-1}, having the ratio $\beta_{s3D}/\sigma_{3D}\approx 1.5\times10^{-5}$ about ten times smaller than in GaAs and InAs. The spin diffusion length in this material is comparable to that in InAs, $\ell_s\approx 2$ μm.

Extrinsic SHE has been also demonstrated in metals, Al (see Ref. 9) and Pt (see Refs. 19 and 20). In Al, $\beta_{s3D}\approx 3\times10^{-3}$ Ω^{-1} μm^{-1}, the ratio of spin Hall and charge conductivities is $\beta_{s3D}/\sigma_{3D}\approx 1\times10^{-4}$, while the spin diffusion length is $\ell_s\approx 0.5$ μm. Therefore, to separate the spin effect from the Ohmic contribution, one needs to fabricate samples with $w\ll\pi\ell_s/[2\ln(\sigma_{3D}/\beta_{3D})]\approx 85$ nm. Although the ratio $\beta_{s3D}/\sigma_{3D}\approx 0.37$ is large in Pt, the observation of the nonlocal effect in this material is hindered by its extremely small spin diffusion length, $\ell_s\approx 10$ nm.[21] We therefore conclude that GaAs systems seem to provide an optimal combination of parameter values for the observation of the nonlocal transport.

IV. EFFECT OF AN IN-PLANE MAGNETIC FIELD

We now analyze the effect of an in-plane magnetic field on the transresistance [Eq. (13)]. In the presence of magnetic field, spin diffusion equation (3) is modified as

$$D_s\partial^2\mathbf{s}-\Xi-\frac{\mathbf{s}}{\tau_s}+[\omega_B\times\mathbf{s}]=0, \qquad (14)$$

where $\omega_B=g\mu_B\mathbf{B}$ is the Larmor precession frequency, $\mu_B=9.27\times10^{-24}$ J/T is the Bohr magneton, and g is the g factor. As we shall see below, the interesting field range is $\omega_B\lesssim D_s/w^2$. Since in this case the variation in spin polarization across the strip is negligible, we can again integrate over the y coordinate and solve a one-dimensional diffusion problem.

For the magnetic field parallel to the x axis, Eq. (14) takes the following form in the Fourier representation:

$$\begin{bmatrix} g(k) & 0 & 0 \\ 0 & g(k) & \omega_B \\ 0 & -\omega_B & g(k)\end{bmatrix}\begin{pmatrix} s_k^x \\ s_k^y \\ s_k^z\end{pmatrix}=-\begin{pmatrix}\Xi_k^x \\ \Xi_k^y \\ \Xi_k^z\end{pmatrix}, \qquad (15)$$

where $g(k)=D_sk^2+1/\tau_s$. For the situation of interest, when only the z component of the source Ξ is nonzero, the solution for s^z is given by

$$s_k^z=-\frac{\Xi_k^z(D_sk^2+1/\tau_s)}{(D_sk^2+1/\tau_s)^2+\omega_B^2}. \qquad (16)$$

Following the same steps as in the absence of magnetic field (notice that the source term Ξ^z is not affected by the magnetic field), we obtain the spin current,

$$J_s(x)=\frac{I\beta_s w}{2\sigma}\mathrm{Re}[q_+e^{-q_+|x|}], \qquad (17)$$

where $q_+=\sqrt{1+i\omega_B\tau_s}/\ell_s$.

The nonlocal response due to the voltage induced by the spin current [Eq. (17)], found as $\delta V(x)=\frac{\beta_c}{2\sigma}J_s(x)$, is

$$R_{\mathrm{nl}}(x)=\frac{\delta V(x)}{I}=\frac{\beta_c\beta_s w}{2\sigma^2}\mathrm{Re}[q_+e^{-q_+|x|}]. \qquad (18)$$

This expression is simplified in the limit of strong magnetic field, $\omega_B\tau_s\gg 1$, by factoring an oscillatory term,

$$R_{\mathrm{nl}}(x)=\frac{I\beta_c\beta_s w\eta}{\sqrt{2}\ell_s\sigma^2}\sin\left(\frac{\eta|x|}{\ell_s}+\frac{\pi}{4}\right)e^{-\eta|x|/\ell_s}, \qquad (19)$$

where $\eta=\sqrt{\omega_B\tau_s/2}$. Compared to the result found in the absence of magnetic field, Eq. (11), the nonlocal voltage $\delta V(x)$ is amplified by a factor of $\sqrt{2}\eta$, decaying on a somewhat shorter length scale $\tilde{\ell}=\ell_s/\eta$.

The dependence of R_{nl} [Eq. (18)] on the in-plane magnetic field is illustrated in Fig. 2. Enhancement of δV at weak

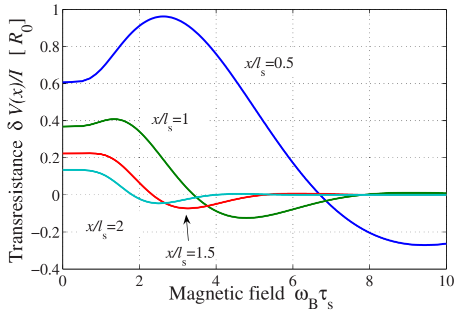

FIG. 2. (Color online) Nonlocal resistance $R_{\rm nl}=\delta V(x)/I_{sd}$, Eq. (18), in units of $R_0=\beta_c\beta_s w/2\sigma^2\ell_s$ vs the in-plane magnetic field for several values of x (see Fig. 1). (Here ω_B and τ_s are the electron spin Larmor precession frequency ω_B and dephasing time.) The nonlocal response *increases* at weak fields, $\omega_B\tau_s\lesssim\ell_s/|x|$, changes sign at $\omega_B\tau_s\sim\ell_s/|x|$, and is suppressed at $\omega_B\tau_s\gtrsim\ell_s/|x|$, simultaneously exhibiting oscillations.

fields, $\omega_B\tau_s\lesssim 1$, is followed by a sign change at $\omega_B\tau_s\sim 1$, and suppression at $\omega_B\tau_s\gtrsim 1$. The zeros of δV can be found approximately for $\ell_s\gtrsim|x|$ from Eq. (19),

$$\omega_{B,n}\tau_s\approx 2\pi^2(n-1/4)^2\ell_s/|x|,$$

with integer $n>0$. [The condition $\ell_s/|x|\gtrsim 1$ ensures that $\omega_n\tau_s\gg 1$, necessary for Eq. (19) to be valid.]

For GaAs, $g=-0.44$ and $\tau_s\sim 10$ ns (see Ref. 7), and therefore the field necessary to observe the oscillations and suppression of $R_{\rm nl}$ at $|x|\sim\ell_s$ is quite weak:

$$B\sim B_*=\frac{\hbar}{g\mu_B\tau_s}\approx 2\ \text{mT}. \tag{20}$$

The transresistance measured at $|x|=\ell_s$ will change sign at the fields $B\approx 11.1$ mT, 60.4 mT,..., decreasing in magnitude as illustrated in Fig. 2.

V. NONLOCAL TRANSPORT IN MATERIALS WITH INTRINSIC SPIN HALL EFFECT

Nonlocal electric transport can result not only from the extrinsic spin scattering mechanisms discussed above but also from the intrinsic spin-orbital effects. Below we present an estimate of the nonlocal effect for a 2D electron gas with Rashba spin-orbit coupling, $H_{\rm SO}=\alpha\hat{z}\cdot(\boldsymbol{\sigma}\times\mathbf{p})$, where $\boldsymbol{\sigma}$ and $\mathbf{p}$ are electron spin and momentum, $\hat{z}$ is the unit normal vector, and α is SO interaction constant. Potential scattering by impurities leads to Dyakonov-Perel spin relaxation with spin diffusion length $\ell_s=\hbar/m_*\alpha$, where m_* is the effective electron mass. In such a system, unlike extrinsic SHE, electric field induces spin density rather than spin current.[11,22] In the narrow strip geometry (Fig. 1) the charge current across the system gives rise to a spin density source $\Xi(x,y)\sim\alpha^3(p_F\tau)^2em_*\sigma^{-1}\hat{z}\times\mathbf{j}_c(x,y)$, where p_F is Fermi momentum, and τ is the momentum relaxation time. Spin transport with such a source, described by Eq. (13) of Ref. 17, exhibits precession induced by drift. This yields a helix-shape distribution[15] with the s_x component oscillating and decaying on the length scale of order ℓ_s.

The spin density, via reverse SHE, creates electric current, $\mathbf{j}'\sim e\alpha^3p_F^2\tau\hat{z}\times\mathbf{s}$, giving rise to transresistance

$$R_{\rm nl}(x)\sim\left(\frac{\hbar\tau}{m_*\ell_s^2}\right)^2\frac{w}{\sigma\ell_s}\text{Re}\ e^{ik|x|}, \tag{21}$$

where $k=k'+ik''$, $k'\sim k''\sim 1/\ell_s$ is a complex wave vector. The transresistance as a function of x exhibits oscillations, similar to those described by Eq. (18).

For a GaAs quantum well with electron density $n=10^{12}$ cm^{-2}, mobility $\mu=10^5$ cm^2/V s, spin-orbit splitting $\alpha p_F=100$ μV, and width $w=0.5$ μm, Eq. (21) gives $R_{\rm nl}\sim 10^{-5}$ Ω, which is somewhat larger than the Ohmic contribution $\sim 10^{-6}$ Ω at $x=\ell_s\approx 3$ μm. However, larger values of the mean free path in quantum wells of ~ 1 μm enhance nonlocality of the Ohmic contribution. This may hinder observation of spin-mediated transport despite a larger value of $R_{\rm nl}$ (cf. Ref. 13).

VI. SUMMARY

In conclusion, spin diffusion in the SHE regime can give rise to nonlocal charge conductivity. A relatively large nonlocality scale, set by the spin diffusion length, can be used to separate the spin-mediated transresistance from the Ohmic conduction effect. In a narrow strip geometry, the transresistance has a nonmonotonic dependence on the external in-plane magnetic field, exhibiting multiple sign changes and damping. Our estimates indicate that observation of the nonlocal conductivity is possible for currently available n-doped GaAs samples. In the intrinsic SHE systems, the nonlocal conductivity can be used to probe interesting spin transport phenomena, such as drift-induced precession[15] and quenching of spin relaxation by quasi-1D effects.[16]

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